Performance optimization and forecasting for nearly full-sky CMB B-mode experiments in the presence of foregrounds

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INTRODUCTION

CMB polarization window onto the physics of the very early Universe → unique insights on the physical laws governing at the highest energies

B-modes detection
- energy scale of inflation
- strong constraint on e.g. total neutrinos mass, and dark energy equation of state

noise goes down with new experiments (more detectors, longer integration time, new technology, etc.)

but **foregrounds are expected to be a fundamental limit.**

Component separation is a critical step in data analysis
OUTLINE

1. METHOD
   definition of our Figures of Merit (FOM)
   maximum parametric likelihood approach to compute residuals
   optimization procedure

2. ILLUSTRATION & RESULTS
   polarized foregrounds modeling
   optimization of future nearly full sky CMB experiment
   extension to some systematics, forecasting

3. CONCLUSIONS
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**METHOD**

**Definition of Figures Of Merit (FOM), quantifying the performance of a CMB B-modes experiment**

**FOM#1 - $r_{\text{stat}}$**  
the $r$ achievable at the $2\sigma$ level

\[
r_{\text{stat}} \simeq 2 F_{rr}^{-1/2} (r_{\text{stat}})
\]

\[
F_{rr} = \frac{1}{2} \sum_{j,j'} \left[ \frac{\partial C_j}{\partial r} \right]^2 + \sum_{l,l'} \frac{(2l + 1) \delta_{ll'}}{2 f_{\text{sky}} C_l^2} \frac{(2l + 1) C_l^{-3} C_{\Delta}^l \delta_{ll'}}{(1 + \sum_{l''} (2l'' + 1) C_{\Delta}^{l''} / C_{\Delta}^{l''})^2} \frac{\partial C_{l'}}{\partial r}
\]

**FOM#2 - $r_{\text{eff}}$**  
level of the residuals

\[
\sum_l^{\ell_{\text{max}}} C_{\ell}^{\text{prim}} (r_{\text{eff}}) = \sum_l^{\ell_{\text{max}}} C_{\ell}^{\Delta}
\]


**FOM#3**  
noise level in the final CMB map

\[\sigma_{\text{CMB}}^{\text{noise}}\]

Your FOM here!  
neutrino mass, SZ, $w$, etc.
How to compute $C_l^\Delta$? Parametric maximum likelihood component separation

$$d_p = B(\beta, \omega) s_p + n_p \equiv \Omega(\omega) A(\beta) s_p + n_p$$

Data $d_p$ is given by the sky signal $s_p$ mixed through the calibration matrix $A(\beta)$ and affected by noise $n_p$. The mixing matrix is parametrized by some $\beta$, and the calibration matrix by $\omega$.

Example in the case of three components: CMB, dust and synchrotron

$$d_p = \begin{bmatrix} d_p(u_0) \\ d_p(u_1) \\ \vdots \\ d_p(u_{#ch}) \end{bmatrix} = \begin{bmatrix} \omega(u_0) & 0 & \cdots & 0 \\ 0 & \omega(u_1) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \omega(u_{#ch}) \end{bmatrix} \begin{bmatrix} A_{CMB}(u_0) & A_{dust}(u_0) & A_{sync}(u_0) \\ A_{CMB}(u_1) & A_{dust}(u_1) & A_{sync}(u_1) \\ \vdots & \vdots & \vdots \\ A_{CMB}(u_{#ch}) & A_{dust}(u_{#ch}) & A_{sync}(u_{#ch}) \end{bmatrix} \begin{bmatrix} s_p^{CMB} \\ s_p^{dust} \\ s_p^{sync} \\ \vdots \\ \vdots \\ \vdots \end{bmatrix} + \begin{bmatrix} n_p(u_0) \\ n_p(u_1) \\ \vdots \\ n_p(u_{#ch}) \end{bmatrix}$$

The mixing matrix is parametrized by «spectral» parameters $\beta$, the calibration matrix by $\omega$. The dependence on hardware parameters: noise per detector, # of detectors, etc.

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estimation of the spectral parameters by maximizing the parametrized Likelihood function

\[-2 \ln \mathcal{L} = \sum_p (d_p - B s_p)^T N^{-1} (d_p - B s_p) + \left[ (\omega - \bar{\omega})^T \Xi^{-1} (\omega - \bar{\omega}) \right]\]

prior term = miscalibration effect

\[\Xi_{ij} \equiv \sigma^{-2}_\omega \delta^j_i\]

e.g. R. Stompor, S. Leach, F. Stivoli, and C. Baccigalupi, MNRAS 392, 216 (2009), [0804.2645].

\[s = \left[ B(\tilde{\gamma})^T N^{-1} B(\tilde{\gamma}) \right]^{-1} B(\tilde{\gamma})^T N^{-1} d\]

second derivative of the Likelihood averaged over noise realizations gives us an estimation of the statistical error on the parameter estimation

\[\Sigma_{ij} \equiv \left\langle \frac{\partial^2 \ln \mathcal{L}_{\text{spec}}}{\partial \gamma_i \partial \gamma_j} \right\rangle_{\text{noise}}^{-1}\]

\[\Sigma^{-1}_{ij} = \text{tr} \left\{ \left[ B_{.,i}^T N^{-1} B (B^T N^{-1} B)^{-1} B^T N^{-1} B_{.,j} - B_{.,i}^T N^{-1} B_{.,j} \right] \sum_p s_p s_p^T \right\} + \left[ (\omega - \bar{\omega})^T \Xi^{-1} (\omega - \bar{\omega}) \right]_{ij}\]

\[N_{ii} \propto \frac{\sigma^2_{det} [\mu K \cdot \text{arcmin}] N_{pix}}{T_{\text{obs}} d_i}\]
residuals computation

\[ \Delta^{CMB} \equiv (\text{estimated sky signal}) - (\text{true sky signal}) = \sum_{k,j} \delta \beta_k \alpha_{k}^{0j} \hat{s}^j \]


\[ \Sigma_{ij} \equiv \left\langle \frac{\partial^2 \ln L_{\text{spec}}}{\partial \gamma_i \partial \gamma_j} \right\rangle_{\text{noise}}^{-1} \]

\[ \alpha_k \equiv \left. \frac{\partial}{\partial \gamma_k} \left[ (B^t(\gamma)N^{-1}B(\gamma))^{-1} B^t(\gamma)N^{-1}B(\hat{\gamma}) \right] \right|_{\hat{\gamma}} \]

\[ C^\Delta_{\ell} \equiv \sum_{k,k'} \sum_{j,j'} \Sigma_{kk'} \alpha_{k}^{0j} \alpha_{k'}^{0j'} \hat{C}_{\ell}^{jj'} \]

FOMs

hardware
**optimization procedure**

Numerical codes use a minimization algorithm for constrained multivariate function optimization procedure.

$$L(d, \alpha) = \text{FOM}(d) - \sum_{i=1}^{m} \alpha_i g_i(d)$$

- e.g. # of detectors in each available channel
- Science or hardware motivated constraints e.g.
  - Total number of detectors
  - Total area of the focal plane
  - CO lines, atmospheric contamination, cost, etc.

**Science motivated FOMs e.g.**

- **FOM#1:** $r \leq 2\sigma$
- **FOM#2:** Level of residuals
- **FOM#3:** Noise level in the CMB map
  - and can be also
  - SZ science, galactic science, etc.
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3. CONCLUSIONS
we only consider thermal dust and synchrotron emission i.e. 3 polarized components with the CMB

- \( A_{\text{CMB}}(\nu_0) \)
- \( A_{\text{CMB}}(\nu_1) \)
- \( A_{\text{CMB}}(\nu_{#\text{ch}}) \)
- \( A_{\text{dust}}(\nu_0) \)
- \( A_{\text{dust}}(\nu_1) \)
- \( A_{\text{dust}}(\nu_{#\text{ch}}) \)
- \( A_{\text{sync}}(\nu_0) \)
- \( A_{\text{sync}}(\nu_1) \)
- \( A_{\text{sync}}(\nu_{#\text{ch}}) \)

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optimization of COrE
1. The CMB is completely swamped by foregrounds.

2. Stretching of the frequency scaling model of the dust.

Optimization in the case of fixed # of channels, fixed central frequencies, fixed area and fixed # of detectors.

**In the following results**

\[ \Omega = 1 \] no calibration error & no other systematics!
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\[
r @ 2\sigma \leq \sum_{\ell} \frac{C^\Delta}{\sum_{\ell} C^{BB,prim}} (10^{-4})
\]

FIG. 10 (color online). Left panel: Results of the FOM#1-based optimization derived in the case of the COREx experiment with a constraint on FOM#2 ($< 10^{-4}$), and using the F06 mask and channels with frequencies below 400 GHz. Right panel: Comparison of the power spectra corresponding to the proposed and optimized versions of the COREx experiment as listed in Table VI and visualized in the left panel. The spectra in blue (mid-level noise spectrum and highest residuals, these latter being depicted with dashed lines) correspond to the cases with the total area constraint. On the other hand, the spectra in magenta (lowest noise level, same residuals as previously) correspond to the cases with the detector number constraint. The foreground residual spectra in both of these cases overlap perfectly in the figure with the magenta curve being invisible.
Optimization of both the # of channels and their central frequencies

FOM#1 | FOM#2 < 10^{-4} & total # of detectors constrained

Optimization of the focal plane results in fewer channels
→ hardware simplification → cheaper experiment
But it all depends on the assumed scaling laws!

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optimal designs are not unique!

→ post processing tests should be applied in order to find reasonable focal plane designs

- detectors number rounding
- low-populated channels removed
- ad-hoc channels addition
- robustness test: effect of some failure rates among the detectors

forecast of future experiments
\[ d_p = B(\beta, \omega) s_p + n_p \equiv \Omega(\omega) A(\beta) s_p + n_p \]

\( \Omega \neq 1 \) → calibration error
spatial variations of \( \beta_{dust} \)
no other potential systematics: detectors bandpass, optical, etc.

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**significance of the residuals**

\[
\sigma_{\alpha}^{-1} = \left[ f_{sky} \sum_{\ell}^{\ell_{max}} \frac{(2\ell + 1)C_\ell^\Delta}{C_\ell^{\text{prim}}(r) + \eta C_\ell^{\text{lens}} + C_\ell^{\text{noise}}} \right]^{\frac{1}{2}}
\]

Fisher error on an overall amplitude of a residuals template

\[ = \text{CMB} + \text{noise} + \alpha \times \text{(residuals template)} \]
What noise level a nearly full-sky experiment should have in order to detect a given $r$ and be safe wrt residuals?

Constraints on $\sigma_{\text{cmb}}$ corresponding to a $1-\sigma$ detection of the residuals on the map level i.e. we solve the equation:

$$\sigma_{\alpha}^{-1}(r, \sigma_{\text{CMB}}) = \sigma_{\alpha}^{-1}\big|_{\text{crit}} (= 1 \text{ in our work})$$
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we introduced a flexible semi-analytic formalism for performance forecasting and optimization of future CMB experiments.

we implemented the formalism in the context of nearly full sky, multi-frequency, B-mode polarization observations (incorporating statistical uncertainties due to the CMB sky statistics, instrumental noise, as well as presence of the foreground signals).


we introduced a measure of the residuals significance and probe its behavior for very low noise levels.

foreground residuals are likely to be a major driver in defining the sensitivity requirements for future experiments, they do not on their own lead to any fundamental lower limits on detectable $r$, at least as long as sufficiently precise frequency scaling models are available.

[JE. & R. Stompor, Astrophysical foregrounds and primordial tensor-to-scalar ratio constraints from cosmic microwave background B-mode polarization observations, Phys. Rev. D 85, 083006 (2012)]
Thank you